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BREAKTHROUGH IN PROBLEM SOLVING

By JAMES GLEICK

A 28-year-old mathematician at A.T.&T. Bell Laboratories has made a startling theoretical breakthrough in the solving of systems of equations that often grow too vast and complex for the most powerful computers.

The discovery, which is to be formally published next month, is already circulating rapidly through the mathematical world. It has also set off a deluge of inquiries from brokerage houses, oil companies and airlines, industries with millions of dollars at stake in problems known as linear programming.

These problems are fiendishly complicated systems, often with thousands of variables. They arise in a variety of commercial and government applications, ranging from allocating time on a communications satellite to routing millions of telephone calls over long distances. Linear programming is particularly useful whenever a limited, expensive resource must be spread most efficiently among competing users. And investment companies use the approach in creating portfolios with the best mix of stocks and bonds.

Faster Solutions Seen

The Bell Labs mathematician, Dr. Narendra Karmarkar, has devised a radically new procedure that may speed the routine handling of such problems by businesses and Government agencies and also make it possible to tackle problems that are now far out of reach.

"This is a path-breaking result," said Dr. Ronald L. Graham, director of mathematical sciences for Bell Labs in Murray Hill, N.J. "Science has its moments of great progress, and this may well be one of them."

Because problems in linear programming can have billions or more possible answers, even high-speed computers cannot check every one. So computers must use a special procedure, an algorithm, to examine as few answers as possible before finding the best one - typically the one that minimizes cost or maximizes efficiency.

A procedure devised in 1947, the simplex method, is now used for such problems, usually in the form of highly refined computer codes sold by the International Business Machines Corporation, among others.

The new Karmarkar approach exists so far only in rougher computer code. Its full value will be impossible to judge until it has been tested experimentally on a wide range of problems. But those who have tested the early versions at Bell Labs say that it already appears many times faster than the simplex method, and the advantage grows rapidly with more complicated problems.

"The problems that people would really like to solve are larger than can be done today," Dr. Karmarkar said. "I felt strongly that there must be a better solution."

American Airlines, among others, has begun working with Dr. Karmarkar to see whether his technique will speed their handling of linear programming problems, from the scheduling of flight crews to the planning of fuel loads. Finding the best answer to the fuel problem, where each plane should refuel and how much it should carry, cuts fuel costs substantially.

"It's big dollars," said Thomas M. Cook, American's director of operations research. "We're hoping we can solve harder problems faster, and we think there's definite potential."

The Exxon Corporation uses linear programming for a variety of applications, such as deciding how to spread its crude oil among refineries. It is one of several oil companies studying the Karmarkar algorithm.

"It promises a more rapid solution of linear programming problems," said David Smith of Exxon's communications and computer sciences department. "It's most important at times when conditions are changing rapidly, for example, the price of crude oil."

If Dr. Karmarkar's procedure performs as well as expected, it will be able to handle many linear programming problems faster than the simplex method can, saving money by using less computer time. But it may also be applied to problems that are left unsolved now because they are too big and too complex to tackle with the simplex method.

For example, A.T.&T. believes the discovery may provide a new approach to the problem of routing long-distance telephone calls through hundreds or thousands of cities with maximum efficiency. Because of the different volumes of calls between different places, the different capacities of the telephone lines and the different needs of users at different hours, the problem is extraordinarily difficult. How the Current Method Works

Mathematicians visualize such problems as complex geometric solids with millions or billions of facets. Each corner of each facet represents a possible solution. The task of the algorithm is to find the best solution, say the corner at the top, without having to calculate the location of every one.

The simplex method, devised by the mathematician George B. Dantzig in 1947, in effect runs along the edges of the solid, checking one corner after another but always heading in the direction of the best solution. In practice it usually manages to get there efficiently enough for most problems, as long as the number of variables is no more than 15,000 or 20,000.

The Karmarkar algorithm, by contrast, takes a giant short cut, plunging through the middle of the solid. After selecting an arbitrary interior point, the algorithm warps the entire structure - in essence, reshaping the problem - in a way designed to bring the chosen point exactly into the center. The next step is to find a new point in the direction of the best solution and to warp the structure again, bringing the new point into the center.

"Unless you do this warping," Dr. Karmarkar said, "the direction that appears to give the best improvement each time is an illusion."

The repeated transformations, based on a technique known as projective geometry, lead rapidly to the best answer. Theory and Practice

"It is very new and surprising - it has more than one theoretical novelty," said Laszlo Babai, visiting professor of computer science at the University of Chicago. "The real surprise is that the two things came together, the theoretical breakthrough and the practical applicability."

Dr. Dantzig, now professor of operations research and computer science at Stanford University, cautioned that it was too early to assess fully the usefulness of the Karmarkar method. "We have to separate theory from practice," he said. "It is a remarkable theoretical result and it has a lot of promise in it, but the results are not all in yet."

Valuable though it may be, like some other discoveries that have come out of the American Telephone and Telegraph Company's research laboratory in the past, the Karmarkar algorithm may not be salable in itself. An algorithm cannot be patented or copyrighted, although specific computer codes can be. Bell Labs is one of several companies that are working on putting it into code.

"A creation of pure thought cannot be protected," said Dr. Graham at Bell Labs. "There will be a whole industry spawned by this as people get a better idea of what's going on."

Dr. Karmarkar, the son and nephew of mathematicians, was born in Gwalior, India, and grew up in Poona, near Bombay. He joined Bell Labs last year after attending the California Institute of Technology at Pasadena and getting his doctorate from the University of California at Berkeley.

News of his discovery has been spreading through the computer science community in preprinted copies of Dr. Karmarkar's paper and in informal seminars. His paper is to be formally published in the journal Combinatorica next month and will be a central topic at the yearly meeting of the Operations Research Society of America this week in Dallas.

Many mathematicians interested in the theory of computer science have long been dissatisfied with the simplex method, despite its enormous practical success.

Business users are most concerned with how well an algorithm performs on the average, and how well it handles the kinds of problems that tend to crop up in the real world. Theoreticians, however, are also interested in the more fundamental issue of whether an algorithm can be guaranteed to solve any problem in a certain number of steps, no matter how awkward or difficult the problem happens to be.

The simplex method, for reasons that were not at all obvious, did much better on the average than theoreticians believed it should. Problems could be designed that frustrated the method, forcing it to step from corner to corner until it had examined all or most of the possible solutions. Yet such problems almost never arose in the real world.

"You had to work hard to produce these examples," Dr. Graham said. But theoreticians continued to wonder whether a better algorithm could outperform the simplex, not just on the average but also in the worst possible cases.

Five years ago, a group of Soviet mathematicians devised a new algorithm, the ellipsoid method, that handled those worst-case problems far better than the simplex method. It was a theoretical advance - but the ellipsoid had little practical significance because its average performance was not much better than its worst-case performance.

The Soviet discovery, however, stimulated a burst of activity on the problem and led to Dr. Karmarkar's breakthrough.

"For a long time the mind-set that the simplex method was the way to do things may have blocked other methods from being tested," said Dr. Richard Karp, professor of computer science at the University of California at Berkeley. "It comes as a big surprise that what might have been just a curiosity, like the ellipsoid, turns out to have such practical importance."

photo of Dr. Narendra Karmarkar (page A19)